Analytic Models of Roll Call Voting Dynamics

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***Abstract*— Roll call modeling is an essential component of analyzing a political system. Current models focus on individual decision-making, and most of them do not take advantage of voting dynamics. Some political systems, such as Ukraine's Verkhovna Rada, are inherently dynamic and should be modeled as such. Therefore, a roll call model is developed from a linear second-order homogeneous differential equation. This model equation is fit to Verkhovna Rada votes from the seventh and eighth convocations. The model determines whether or not bills will reach the passing threshold with 77% and 85% accuracy for the seventh and eighth convocations, respectively. It is shown that the dynamic legislative model is slightly less accurate than a neural network, but it is significantly more interpretable. This interpretability is vitally important, as it is what makes models meaningful beyond their predictive power. It is found that bills sponsored by the president show quantitatively different behavior than ordinary bills and the ordinary bills are largely decided in the first two votes. Furthermore, our models have intuitive theoretical implications, some of which are back by prior work. The models suggest that MPs are less willing to change their vote on bills as iterations increase and they are more sensitive to change the public opinion if the bill is sponsored by the president. While the majority of bills are modeled well, about 25% of votes have greater than 10% error. Investigation of these votes indicates that some votes may be impossible to predict without a more complex model which incorporates contextual information. Finally, the information from a bill's first two votes is also leveraged through a vote switching network. This directed network gives insight into who sends the most powerful signals and who follows them. An ensemble of centrality members is then used to identify the legislator's most influential members.**

***Index Terms*— Differential modeling, roll call analysis, roll call dynamics, Verkhovna Rada.**

I. Introduction

ROLL call voting has been extensively studied over the last 50 years to better understand legislative bodies and the larger political systems they belong to. Roll call data can be used to determine the level of cooperation within the government, underlying political factions, and which parliamentarians are most influential. In this paper, we seek to demonstrate the potential of using a new theoretical framework for a roll call analysis: viewing the legislative process as a dynamical system. We use this framework to answer two simple questions about the legislature: "will the next vote on a bill be successful?" and "who is influential in the parliament?"

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In most prior work, the focus has been on modeling individual decision-making. In order to model a legislator's thought process, as much contextual information as possible is needed. Often, the analysis of a large number of bills is needed to determine political dimensions. Then, information about each legislator is needed to find their position in the political dimensions. Finally, the position of a legislator relative to a bill in this political space is likely to determine their vote.

While this method is intuitive and generalizable, it does not take into account temporal aspects of roll calls. Some legislative systems, such as the Ukraine's Verkhovna Rada, are inherently dynamic and should be modeled as such. In systems such as these, bills are voted on several times before passing. The *change* between votes, then, provides rich information about the legislative system and the parliamentarians with the most influence. Additionally, prior voting information can be leveraged to predict the final voting outcomes for the bills.

Some recent work has used game-theoretic models to get at the temporal problem through turn taking. Even with this improvement, individual decisions are still being modeled, and therefore, large amounts of contextual information are still needed. If only the bill outcome is of interest, modeling individual decisions is overcomplicating the process. Dynamic modeling allows each bill to be represented by a single variable, the percentage of votes for, and does not require contextual information beyond prior voting outcomes.

Here, we develop a dynamic model of the legislative process. Drawing on theory from state-space identification, the voting data are used to determine a single model for the entire legislative body. Every bill's voting trajectories can then be derived. While complex high-order models could be constructed, our goal is to use a workable model that is also interpretable, so that we may draw conclusions about the general roll call process. We show that a simple low-order model is not only interpretable but also powerful in voting prediction. Additionally, no contextual information is necessarily needed for prediction or analysis making it an easy tool to use in the future real-time voting prediction.

While modeling the legislature as a dynamic system, individual-level information is lost. To recover this, we introduce the vote switching network. Motivated by our results from the dynamic analysis, this network attempts to capture individual influence through the iterative roll call process. We analyze this network to find influential MPs and parties for both convocations.

In Sections II to IV, we provide the background information regarding the Verkhovna Rada and our data, walk through the prior work in roll call analysis and state-space identification, introduce the dynamic model, and analyze its effectiveness. Then, we introduce the vote switching network and discuss

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TABLE I
BILL SUMMARY STATISTICS OF BILL OUTCOME VARIABLE, υ

Convocation	Subject	Bills	Max Iterations	Predictable Points	Committees	Mean	Std. Dev.	Min	Max	Median
7	All	231	15	498	25	45.42	12.36	1.253	80.58	45.93
7	Parliament	187	15	404	25	46.05	12.31	2.714	80.58	46.03
7	Government	39	15	85	17	42.30	12.52	1.253	75.78	43.84
7	President	5	6	9	3	45.95	8.234	25.26	64.02	47.60
8	All	577	15	1790	22	39.25	8.718	2.667	65.334	41.34
8	Parliament	406	15	1257	22	39.49	8.437	2.667	65.34	41.33
8	Government	133	15	437	20	38.64	9.135	9.143	58.286	40.76
8	President	38	15	96	10	38.86	10.19	4.952	63.429	41.43

its analysis. Finally, we state our limitations, conclusions, and avenues of future work.

II. VERKHOVNA RADA

A. Voting Procedure

Under the Verkhovna Rada's legislative process, the same bill is voted on several times by the same set of legislators. Technically, a bill needs three successful votes, defined by 225 or more votes "for," to be put into law. In reality, however, there are many votes calling for special procedures, such as a vote for an expiated process. Our data set does not distinguish between these votes, so we treat every vote iteration uniformly, and attempt to model the percentage of votes it receives. From this value, we can determine if it is above or below the passing threshold, but we cannot determine if the bill was actually put into law.

This voting process introduces a time component that commonly studied legislature's lack. The United States Congress, for example, only requires one vote per legislative body. While not often studied, the component can be extremely useful when attempting to predict bill outcomes. Furthermore, it can be used to track parliamentarians influence and the relationships between the parties. These capabilities are demonstrated in the rest of the work. While the voting system is uncommon, it is not unique. For example, Russia, Estonia, and Philippians all require preliminary votes on their bills.

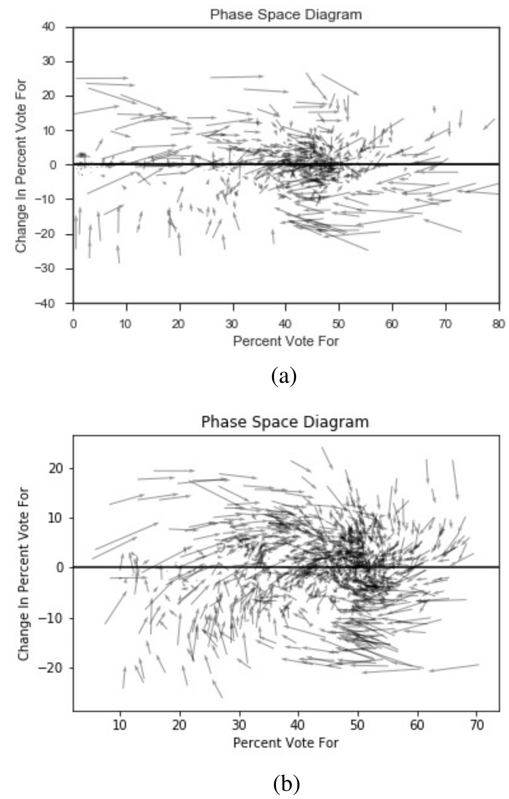

Fig. 1. Phase portraits of percentage of votes for. (a) Convocation 7. (b) Convocation 8.

B. Data

The Verkhovna Rada is broken up into convocations or a legislative term of office. Typically, a convocation spans two years. Here, we analyzed convocations 7 and 8. The context surrounding these convocations is drastically different due to the Ukrainian revolution of 2014. This revolution happened in the middle of convocation 7, and therefore, we expect that the voting is significantly more polarized than that of convocation 7. It is of interest, then, to see if these contextual differences can be overcome with our model.

We only focus on the roll call data and basic information about the bills. For each iteration of each bill, the MPs were listed along with the vote that they cast. Additionally, each bill has a label indicating the sponsoring subject (parliament member, cabinet member, or the president) and the sponsoring committee. Summary statistics of υ broken out by convocation and by subject is shown in Table I. These factors are modeled individually to analyze their effect on bill behavior.

We are primarily interested in bill outcomes, which only depend on the percentage of votes a bill receives. Hence, the data were collapsed into the percentage of votes "for" each iteration or υ. Since this paper investigates υ as a dynamic variable, its phase portrait is shown in Fig. 1. Phase portraits show the relationship between a variable and its derivative, so a structured diagram justifies the use of a differential model. Here, we see an inward spiral for both of these equations. This is seen by picking an initial point and following the arrows at that point. In terms of theory, the difference between a spiral inward (toward the x-axis) and a spiral outward (away from the x-axis) is huge. Inward spirals are the characteristic of stable models, meaning that long-run solutions are stable. In our context, this means that bills will eventually reach a stable amount of votes for or that less and less MPs are willing to change their vote. Fig. 1 shows that negotiations, discussions, and trading favors among members of the Rada result in a convergence process, such that those bills that pass are likely to do so by a slim margin. Since voting sends a strong signal on position, choosing one of the nonvote options may be safer even when an MP favors the bill. In countries

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where the state controls the legislature or where consensus is encouraged, we would expect to see the convergence process, resulting in a higher margin of passing for those bills that are voted to pass. Furthermore, it seems that we may be able to find the long-run result of bills after only two or three iterations. This is the general goal of this work.

III. Related Work

Traditionally, roll call voting has been analyzed spatially. In the simplest case, both legislators and voting choices are placed on a single political dimension. Based on a legislator's position relative to a choice, a probability of their vote "for" is determined. An example of a simple spatial model is given in Weissberg's dissertation [23]. Closely related to this is the concept of a Guttman scale, in which choices are ordered such that if a legislator agrees with one choice, they will agree with all preceding choices.

MacRae acknowledged that a 1-D political space is insufficient and proposed a method for calculating other dimensions [14]. Poole, Clausen, and others have used these political dimensions as the basis for legislative decision-making [3], [17]–[20]. This became even more popular through the creation of NOMINATE, which expanded MacRae's "crucial gap" between defining the dimensions and calculating the distance between them. All of the models rely on voting history and bill source information to place entities spatially. Additionally, some of them even need to segregate the data by the official party or to pare down the resulting dimensions to get meaningful results [3], [14]. Furthermore, scaling to obtain meaningful dimensions, Bayesian inference methods have been investigated [9], [13].

Most of these models were created with the United States Congress in mind. In application to the U.S. Congress, the concept of "dynamics" usually refers to how opinions or other legislative characteristics change in time. Dynamic spatial models have been investigated through the use of Markov chain Monte Carlo [16]. More recent work by Duggan and Kalandrakis [6] and Kalandrakis [10] has looked into dynamics through game-theoretic models of roll call votes and of the political status quo. A game-theoretic model of legislative voting still attempts to model individual decisions and thus requires contextual information. In this paper, however, we are not looking at long-term individual behavior, but rather short-term group behavior. To the best of our knowledge, a dynamic model of an overall legislative system capable of predicting voting outcomes on bills has not been attempted.

Differential equations are the prevailing model used to describe systems with a temporal component. Differential models are that the dependent variable is a function of its own derivative. There are many applications in which the change of a variable is just as significant as the variable itself. Physics, for example, models systems through acceleration, which is the position's second derivative. Fields, such as physics, have laws that can be used to derive differential models' parameters. In other domains, such as roll call analysis, these laws are not available, and thus, parameters must be estimated from data. This process is known as state-space identification or system identification [8]. The goal of this paper is to draw on this literature while creating interpretable models. We believe that analytic models with a relatively few terms are most interpretable, and therefore, we constrain our results to meet this criterion. Interpretability will be further discussed in Section IV-D.

As discussed by Brewer *et al.* [4], there are many ways to estimate linear model's parameters, such as shooting, b-spline collocations, and methods based on derivative estimation [1], [5], [7]. The first two models minimize the error between the data and the model's output. Although this is intuitive, it relies on the numerical integration of the model. The derivative-based method is named for its use of estimated derivatives to fit parameters rather than numerical integration. This scheme allows us to define the model such that analytic solutions are guaranteed. While using b-spline collocations could result in an analytic solution, a long series of polynomials will be significantly less interpretable than the solutions of a simple second-order model. As such, we will proceed using a derivative estimation method of finding a second-order linear model, fully described in Section IV-A.

This paper expands on preliminary analysis of Ukrainian voting dynamics [15]. The preliminary work used 78 bills from convocation 8, while here we analyze 577 bills from convocations 8 and 231 bills from convocation 7. This allows for comparison between convocations with large contextual differences. Additionally, a more appropriate minimization scheme has been implemented. In the preliminary work, we also found evidence that legislators responded differently to presidential bills rather than standard bills. There is a prior work studying whether or not MPs face consequences for voting against their party or political leaders. Canes-Wrone *et al.* [2] find that there are benefits to voting against their party, such as increased odds of retaining their position. Furthermore, Kauder *et al.* [11] failed to find evidence that MPs are punished for voting against their party in Germany. We further investigate this question by studying the difference in MP behavior regarding presidential bills and again conclude that MPs are quicker to conform with the greater legislature when bills are sponsored by the president.

Finally, we expand on previous work by the introduction of the vote switching network. While the analysis of the aggregate behavior of the legislature is a powerful and insightful method, the ability to analyze specific MPs is lost. Through the vote switching network, however, we leverage our group-level insights to create an influence network at the MP level. The network, then, is analyzed with centrality measures to find powerful MPs.

IV. Roll Call Votes as a Dynamic System

A. The Model

In contrast to previous work focusing on individual decision-making, we propose to model the legislative body as a single unit. This simplification still gives insights to bill outcomes without the complexity of modeling individual MPs. Under this framework, bills are instances of a larger process, and all follow the same set of rules. We also introduce the vote switching network in Section V to combat the issue of lost granularity.

The legislature is modeled together by collapsing the problem to a single variable, v, the percentage of votes in favor of a bill. Our hypothesis is that this variable's behavior is a function of its iteration, i, meaning that given knowledge of previous votes on a bill, we can make predictions about future iterations. A preliminary test of this hypothesis is completed through the visualization of the variable's phase space, as shown in Fig. 1. In the phase diagram, we see a spiraling structure, indicating that initial votes may ultimately indicate their fate.

This process could potentially be modeled with a Markov Chain, but it would assume that only the previous vote matters. Using an ordinary differential equation (ODE), however, the vote's *change* matters. This model is more intuitive; if a bill is gaining support from one iteration to the next, it will likely continue to do so. Furthermore, a spiraling structure in phase space may be captured using an ODE. As such, we will proceed with that model in mind. We propose a simple second-order homogeneous equation of the form

$$A\frac{d^2v}{dt} + B\frac{dv}{dt} + C = 0. \quad (1)$$

This model is proposed for two reasons. First, it ensures an analytic solution. Analytic solutions are significantly more interpretable (see Section IV-D.) Second, this is the simplest possible model that may capture the spiraling phase space. Success with this model shows the power of the dynamic approach, as a more complex differential model can always be used. A more complex model may include higher order or nonlinear terms and/or should be nonhomogeneous.

While votes are discrete, our model assumes that the variable v is continuous in time t, instead of discrete in i. To predict votes, then, we say that votes occur at regular intervals of an unknown parameter, α, i.e., votes are predicted only at regular intervals, t_i, where $t_i = \alpha * i$. The interval, α, can either be taken into account when calculating the derivatives or in the model equation itself. Since α is unknown, it is inserted into the model equation directly. Typically, the highest order term does not have a coefficient. Thus, after normalization, the final model equation becomes

$$v'' + \alpha C_{v'} v' + \alpha^2 C_v v = 0. \quad (2)$$

From here, the equation is solved analytically. If the characteristic polynomial has real roots, the solution form will be

$$v(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}. \quad (3)$$

With complex roots, the solution becomes

$$v(t) = C_1 e^{r_1 t} cos(r_2 t) + C_2 e^{r_1 t} \sin(r_2 t). \quad (4)$$

It is possible that the characteristic polynomial has a repeated real root, but this is highly unlikely since 2 is numerically estimated.

Regardless of the form, the first two votes are fit exactly for each bill through the coefficients C_1 and C_2. This is only possible because 2 ensures an analytic solution. Without an analytic solution, simulation from the first vote would require an estimate of the initial derivative, leading to an incorrect second vote despite having knowledge of its outcome.

Finally, there are two main ways that the model can be used. The first is to fit parameters to the first two votes of a bill and use the resulting function to predict many votes ahead. This method will be referred to as the "simultaneous prediction" method. The second method is to only predict one vote at a time. The initial conditions of the model are updated to be the two most recent votes by refitting the C values. This method will be referred to as the "updated knowledge method." Since this method seems more practical, it will be the primary method used for analysis.

The use of a differential model may not seem much different from a standard regression, but the implications are significant. With this approach, a *single equation* [see (2)] captures the behavior of the entire legislature. In this model, bills are not independent events but instances of a larger process, and all follow the same set of rules.

B. Fitting Parameters

To fit the model parameters, an objective function was defined based on (2)

$$F = \sum_{b=1}^{N} (v_b'' + \alpha C_{v'} v_b' + \alpha^2 C_v v_b)^2 \quad (5)$$

where b represents the individual vote and N is the total number of votes that have a calculable double derivative. This equation is simply the model equation calculated over all the data points available, squared. Squaring the values allows minimization of F to converge near 0 rather than blowing up with large negative coefficients. Thus, minimizing this function over the coefficients α, $C_{v'}$, and C_v leads to the model equation that best fits the data.

This objective function relies on the second derivative of the voting signals, which are originally discrete. To estimate these derivatives, we fit each set of three consecutive votes with a quadratic function and use the derivatives of the quadratic function. While prior work suggests the use of splines (see [5]), this method requires only three data points, and the same number is required to predict one vote.

Note that the objective function sums over all instances of votes with a calculable double derivative, which may or may not include multiple votes from the same bill. Using multiple points from a single bill is a valid operation, because it is the only way to ensure that each individual bill signal follows the model equation.

Since the objective function basically sums over the data, it is filled with local minima. To find a global minimum, the differential evolution algorithm was used [21]. This stochastic algorithm finds the global minimum of a restricted search space which we defined as: $\alpha \in [0, 2], C_{v'} \in [-5, 5], C_v \in [-5, 5]$. Since alpha only adjusts discrete time to continuous, only positive values are considered. The resulting coefficients are then used to derive bill trajectories.

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TABLE II

FIT MODEL PARAMETERS FOR (3) AND (4). MEAN AND MEDIAN REFER TO THE PREDICTION ERROR. FRAC. REFERS TO THE FRACTION OF VOTES CORRECTLY PREDICTED BASED ON THE MAJORITY THRESHOLD. IN ALL CASES, THE UPDATED KNOWLEDGE METHOD WAS USED. NOTE THAT ONLY PRESIDENTIAL MODELS HAVE REAL ROOTS OR DIFFERING SIGNS

Convocation	Model	Roots	r_1	r_2	Mean	Median	Frac.
7	All	Imaginary	-0.045	-0.284	14.0	7.57	0.770
7	Parl.	Imaginary	-0.037	-0.265	13.9	7.51	0.765
7	Cabinet	Imaginary	-0.127	-0.338	13.2	7.13	0.8
7	Pres.	Real	3.27	0.086	328.0	202	0.333
8	All	Imaginary	-0.015	-0.193	10.2	6.16	0.850
8	Parl.	Imaginary	-0.013	-0.202	9.57	5.63	0.864
8	Cabinet	Imaginary	-0.062	-0.155	11.0	6.98	0.826
8	Pres.	Imaginary	0.093	-0.173	14.0	10.1	0.77

C. Model Results

To evaluate the models, we call the absolute difference between the model prediction and bill outcome the "error," which will be used throughout the results. Additionally, the model can be used as a binary classifier determining if a vote will be above or below the passing threshold. Table II displays the mean and median error and the fraction of votes classified correctly for each model using the updated knowledge technique.

1) All Data: The "all data" model was found by fitting parameters to the entire set of bills, ignoring any data on the subject and committee. Convocation 8 is modeled more accurately than convocation 7, with 6.2% and 7.6% median error, respectively. Also, 77% and 85% of votes were classified as passing or failing for convocations 7 and 8, respectively. This result is expected from the phase space diagram in Fig. 1. While both convocations show a spiraling behavior, convocation 7's shape is less regular. For bills with less than 40% of votes "for," bills are either moving vertically (below the horizontal axis) or horizontally to the right (above the horizontal axis.) In the same areas in convocation 8, the bills follow a smoother clockwise curve, in which the model can better capture. Intuitively, this means that votes changed more quickly from iteration to iteration in convocation 7 than in convocation 8.

A more detailed view of the "all data" model error is shown in the box plots displayed in Fig. 2. The errors per vote iteration are displayed for each convocation. Overall, the error distributions are similar from iteration to iteration, with the exception of iterations 10 and 13 in convocation 7, as well as iteration 13 in convocation 8. These iterations have a noticeably higher error. For convocation 8, about 75% of predictions have 10% or lower error. For convocation 7, it is about 15% or lower error. Both convocations have many outlier votes with an extremely high error, which will be discussed in Section IV-E

2) Models by Subject: Experts have suggested that presidential bills behave differently than standard parliamentary bills. It is possible that these bills are pushed through faster since they are higher priority or that parliamentarians vote differently out of fear of the president. To capture this, bills were modeled separately based on their initial subject.

In convocation 7, subject-by-subject models improved median accuracy for the parliamentary and cabinet bills but resulted in an unusable model for presidential bills. These bills

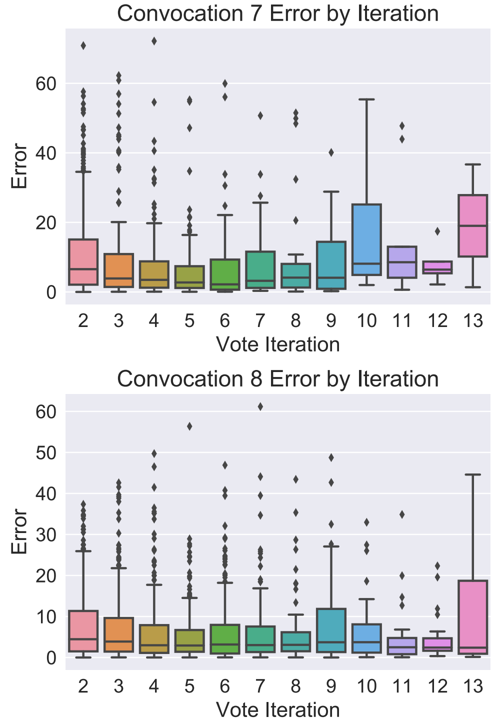


Fig. 2. Box plots for the absolute error from the "all data" method. Low median error is seen, but there tails with a very high error.

were only to convert to a solution with real eigenvalues causing predictions to blow up quickly and produce meaningless results.

Similar results were found for convocation 8. Parliamentary and cabinet bills had minor changes in their error distributions. While the convocation 8 presidential model has imaginary eigenvalues and reasonable output (a median error of 10.1% and 77% classification accuracy,) it is unstable. This instability is discussed in detail in Section IV-D1.

3) Models by Committee: Finally, it was hypothesized that the remaining 25% of bills were modeled poorly due to the content of the bills themselves. We modeled the content of bills through the committee they were sponsored by. However, individual committee models were unable to accurately predict these bills, proving the hypothesis false. Splitting the data by subject and committee was also uninsightful, especially considering that this did not further segment the presidential data.

4) Neural Network Comparison: For comparison, a neural network approach was taken to model the data. For simplicity, this is only done in direct comparison to the "all data" model. The data were split randomly with one third being used for testing and two thirds for training. Then, a grid search was used to find the model with the highest accuracy. This search included four activation functions, "tanh," "identity," "logistic," and "rectified linear unit," three L2 regularization penalties, 0.00001, 0.0001, and 0.001, and four hidden unit sizes, 10, 50, 100, and 500, for a total of 48 possible models, all with 1 hidden layer.

The best model was found to be a 100-hidden-unit network with rectified linear unit activation functions. The L2 penalties were 0.001 and 0.0001 for convocations 7 and 8, respectively.

The median test error for convocation 7 was 4.98%, with 25% of votes having $\geq$ 12.4% error. The fraction of votes classified correctly was 0.757. For convocation 8, the median test error was 3.36%, with 25% of votes having $\geq$ 8.7% error. The fraction of votes correctly classified was 0.925. In both convocations, the neural network approach led to lower median error predictions, but it still had 25% of votes with a relatively high error. In convocation 7, our method better-classified votes as above or below the passing threshold. In convocation 8, however, the neural network model was a better predictor.

D. Model Discussion

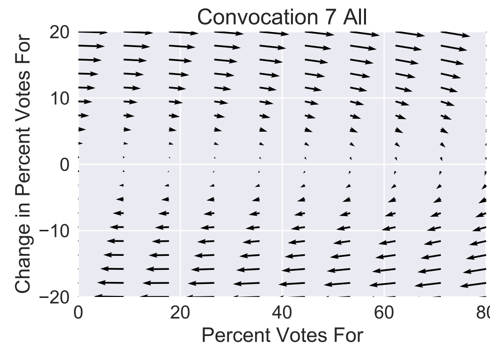
Generally, the spiraling behavior of the bills in phase space shows an underlying structure to the legislature that is mostly independent of a bill's actual content. This structure is stable, that is, bills on either side of 50% approach neutrality, slow down, and stop. This indicates that MPs are willing to show strong support/opposition to a bill in the early votes. In the following iterations, some MPs change their vote. As the iterations increase, less and less MPs are willing to change their vote, and a stable outcome is reached.

Which side of 50% bills stop on likely determines its success or failure. The key point of this underlying process is that the second vote determines a bill's initial trajectory, which tends to determine the trajectory over its lifetime, allowing it to be modeled and predicted.

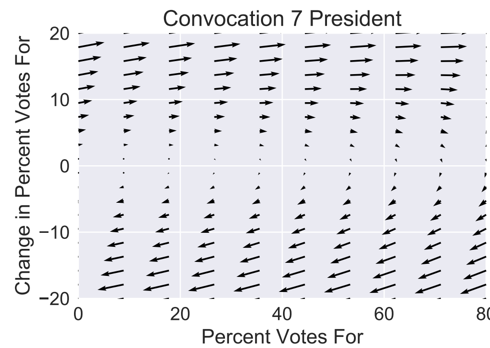
1) Model Behavior: As experts suggested, the behavior of presidential bills is different from that of parliamentary bills. Since the presidential bills have eigenvalues of different signs (see Table II), the model is unstable. The system's instability is visualized in Fig. 3(b), which displays the presidential model's phase portrait for convocation 7. Where the "all data" model, shown in Fig. 3(a), has a spiral in behavior, the presidential model spirals outward. Again, the difference can be seen by picking an initial arrow on the diagram, following its direction, and adjusting when encountering other arrows. The spiral outward leads to unreasonable predictions quickly, which is why it is unsuccessful if applied to bills with many iterations.

For bills with a few iterations, however, this model may be explained intuitively through fear of the president. On the first vote, MPs may vote as they wish. By the second vote, they know what everyone else has voted and change their votes accordingly. On the third vote, however, MPs can see if the bill is gaining popularity or losing it. If it is gaining popularity, and thus likely to pass, many MPs also agree to vote "for," as to not unnecessarily be on the wrong side of the president's bill.

A more intuitive way of understanding the models is by visualizing their predictions for a single bill. Fig. 4 shows the trajectories of models from both convocations given a single bill. Fig. 4(a) shows each model's prediction given votes from a bill in convocation 7, and Fig. 4(b) shows the same but for a bill in convocation 8. Both the models follow a near-linear trajectory from the two preceding votes. However, each model gives slightly lower than an exact linear trajectory, with the convocation 8 model dipping further than the convocation 7 model. Hence, the models are sensitive to large changes in a single vote. This is shown in iterations 6–8 in Fig. 4(a). The large dip from vote 6 to 7 causes both models to predict a very low (effectively zero) vote for iteration 8, but in reality,

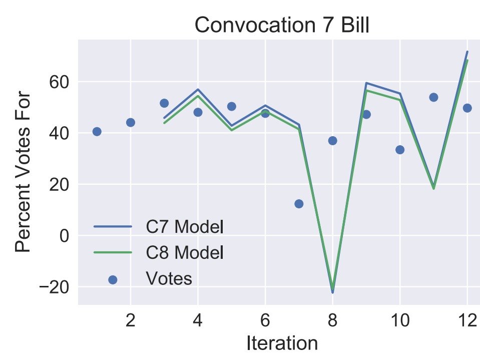


(a)

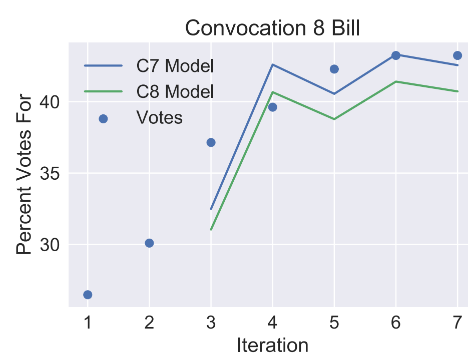


(b)

Fig. 3. Phase portraits of (a) convocation 7 all data ODE and (b) convocation 7 presidential ODE.



(a)



(b)

Fig. 4. Modeled trajectories compared to the actual votes for (a) convocation 7 bill and (b) convocation 8 bill. "All data" models were used for both figures. Blue line: convocation 7 model. Green line: convocation 8 model.

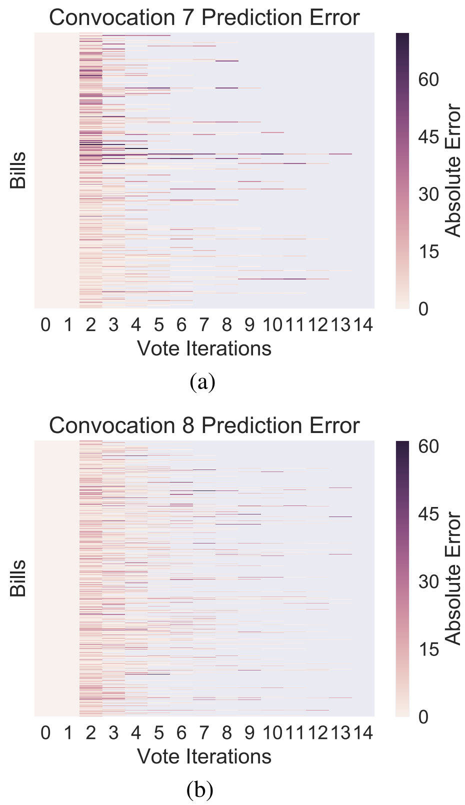


Fig. 5. Heat map for bill errors. Columns: vote iteration. Rows: different bills. (a) Convocation 7. (b) Convocation 8.

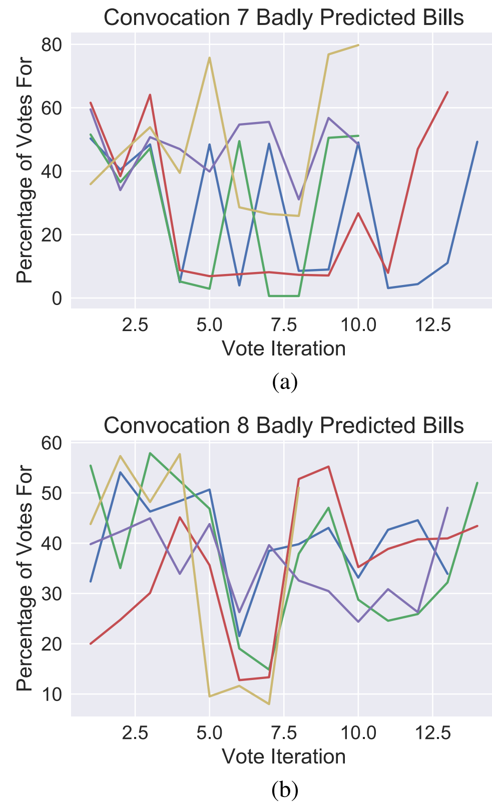


Fig. 6. Trajectories for the top five worst modeled bills in terms of the number of iterations with > 10% error. (a) Convocation 7. (b) Convocation 8.

the bill gained popularity after the dip. Since convocation 7 has more of these irregular voting patterns, this modeling technique performs worse overall, as seen in Table II.

It is important to note that the mostly "straight-shot" outcome of the model is an artifact of our choice of method rather than the model ODEs. If votes were predicted using the simultaneous prediction method, iterations greater than 3 would no longer seem like a linear prediction.

E. Poorly Modeled Votes

A quarter of the votes remain elusive, with errors above 10% for each of the models. Two potential sources of this error are the bills themselves and the vote iteration. The error heat map in Fig. 5 is used to distinguish between the two. If the particular bills were hard to predict, the heat map would have a few dark rows. If the critical iterations were harder to predict, the heat map would have a few dark columns. It appears that the very high error votes happen sporadically, but there are bills with high error overall.

The trajectories of bills with the highest average error are plotted in Fig. 6. These trajectories are all filled with large, seemingly random jumps. One bill in convocation 7 falls from over 60% to 10% in a single iteration, has eight votes with a very few votes "for," and finally climbs back up over 60%. The simple differential models used here are simply incapable of capturing such eradicate behavior. In fact, it is hard to see how any model could predict all of the trajectories in Fig. 6, knowing only the first two votes. Even the 100-hidden-unit neural networks trained on the data had a high error for about 20% of the votes. It seems that while higher order models could still be used to increase accuracy, the use of contextual domain knowledge is the only way to predict some votes.

Looking at the bill's content reveals that the bills in convocation 8 covered topics, including pension tax, college funding, natural gas regulations, orphan care, and court fees. Convocation 7 bills covered topics, including the gas transportation system, corporate tax, administration reform, financial bill procedure, and food market regulations. In both cases, the bill topics vary dramatically. To complicate things further, the divisive points in bills are often cleverly obscured or hidden, further emphasizing the need for expert legal analysis.

V. Vote Switching Network

Section IV illustrated the power of viewing roll calls as a dynamic process, specifically using the change from the first to the second vote. It follows that the parliamentarians who are driving the change in the first two votes are highly influential. As such, a network can be constructed from the change in the first two votes, and centrality measures can be used to find influential parliamentarians.

The vote switching network is created from the first two votes on each bill. If an MP changes their vote, an edge is added from them to whoever casted this vote initially, that is, if an MP changes their vote from "abstain" to "for" they are switching their vote to those who initially voted "for," and the network will reflect that. The weight of the edge is proportionate to how rarely the vote cast is. Since votes "against" are very rare, switching to match someone's

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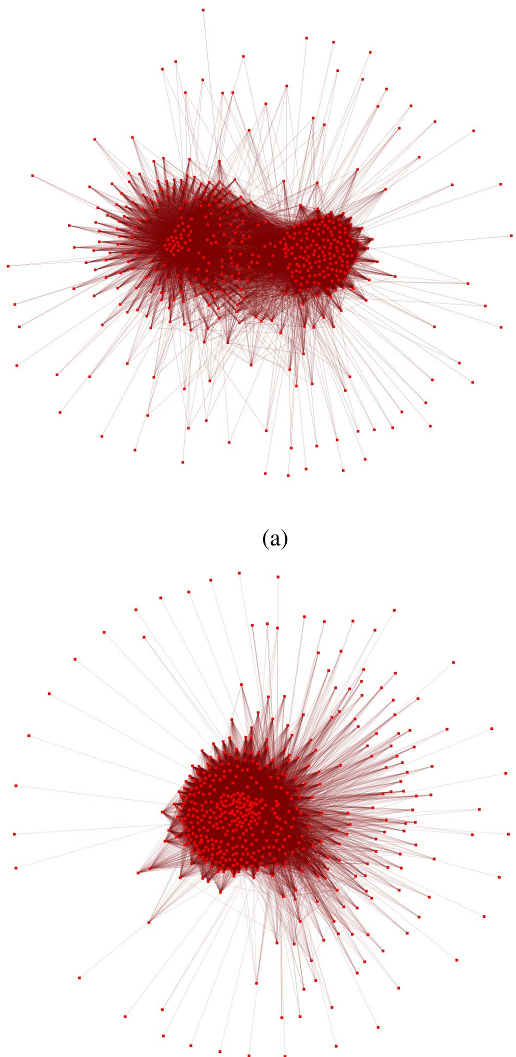
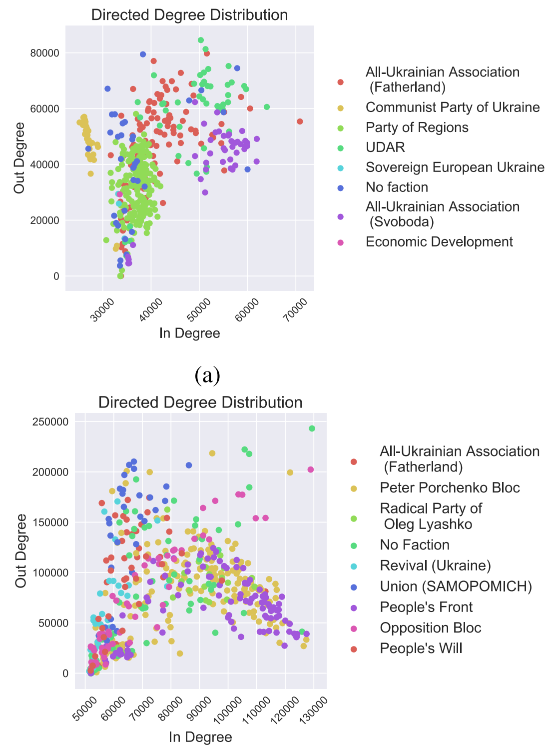


(a)

(b)

Fig. 7. Vote switching network visualized. Links with a weight below the mean + one standard deviation are not shown. Modularity is 0.294 for (a) Convocation 7 and 0.080 for (b) Convocation 8.



(a)

(b)

Fig. 8. In- and out-degrees are shown for each MP, colored by official party affiliation. (a) Convocation 7. (b) Convocation 8.

"against" vote is weighted heavily. The edges are summed over all the bills to obtain the final weighted and directed vote switching network. The networks for both convocations can be seen in Fig. 7.

The vote switching network for convocation 8 shows more of a solid core than that of convocation 7. Convocation 7 seems to have almost two separate cores, with common members between them. This may suggest more tension within the legislature, as we would expect given the political revolution occurring at the time. The modularity for the convocations is 0.294 and 0.080 for convocations 7 and 8, respectively, which also highlights this difference.

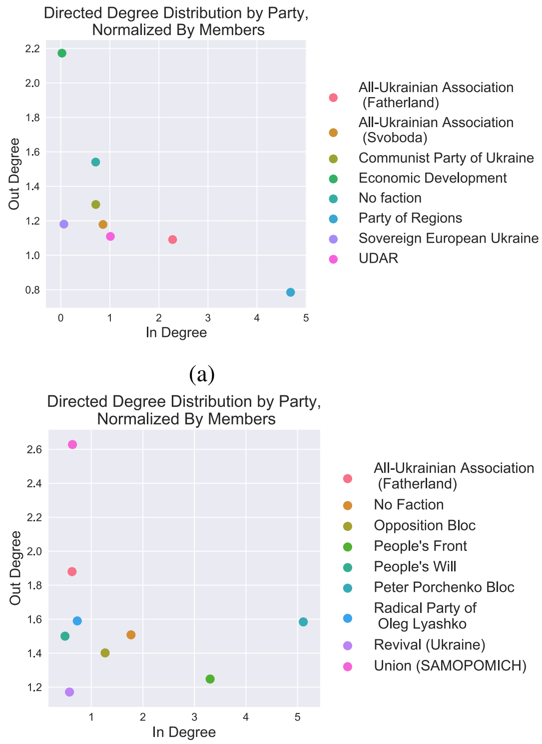
Both networks were found to be disassortative, with coefficients -0.012 and -0.016 for convocations 7 and 8, respectively. This result is intuitive from the motivating theory of the legislature; a disassortative vote switching network implies that MPs who often switch their vote do so to match those that do not switch their vote. This gives credence to our underlying assumption; influential MPs are less likely to switch their vote than others.

At an individual level, an agent's degree can be used to infer their influence. The in-degree and out-degree are plotted by a parliamentarian in Fig. 8. It is expected that a highly influential individual would have many people switching their votes to match theirs and would not change their votes as often as others. Thus, the closer to the bottom right-hand corner in Fig. 8, the more influential the MP is likely to be.

In convocation 8, influential members falling in the bottom right of Fig. 8 come from the Peter Porchenko Bloc and People's front, the two dominant parties. In convocation 7, however, parliamentarians with high in-degree also have high out-degree, so the clearly influential portion of the plot is empty. We can see that many of the individuals with high in-degree are coming from one of the All-Ukrainian Associations.

To analyze party relations directly, the vote switching network is aggregated by party to obtain the directed party–party network. The out-degree of each party is normalized by its number of members. Both the degrees are plotted in Fig. 9. Within-party votes, or self-loops, are not considered in the distributions in order to only measure the influence between the parties, not within them.

In these diagrams, the parties fall more neatly along an "influence line" connecting the top left to the bottom right. In convocation 7, the Economic Development party is the least influential, and the Party of the Regions is the most. While convocation 8's layout is less clear, the Peter Porchenko Bloc is the most powerful, and Samopovich is the least.



(b)

Fig. 9. In- and out-degrees are shown by party, with out-degree normalized by the number of party members. (a) Convocation 7. (b) Convocation 8.

This diagram can also be used to find insular parties, which have low in- and out-degrees. Convocation 7's most insular parties are Sovereign European Ukraine and The Ukrainian Democratic Alliance for Reform (UDAR). Revival is the most insular in convocation 8.

More quantitatively, the vote switching network can be analyzed using centrality measures to find the most influential parliamentarians. The results from this analysis are shown in Table III. Two of the leaders from the convocation 7 vote switching network belonged to UDAR, while the third was part of the All-Ukrainian Association (Fatherland). In convocation 8, UDAR merged with the Peter Porchenko Bloc. Although Fatherland remains in convocation 8, Brichenko Igor Vitaliyovych still switched to the Peter Porchenko Bloc, putting the top three influential members of convocation 7 in the same party during convocation 8. The most central members of the convocation 8 vote switching network, however, are not associated with the Peter Porchenko Bloc. Instead, two have no official party and one is a member of the Opposition Bloc, the leading force against the presidential party. Future work may involve domain experts diving deeper into the backgrounds of key parliamentarians to better understand why they are central in this network and the significance of the shift in power to members without an official party affiliation.

VI. Threats to Validity

Perhaps the biggest threat to validity comes from the data itself. Without contextual information about the individual votes, it is impossible to distinguish between a vote to put a bill into law and a vote for special proceedings on a bill. For example, a vote may be cast on whether or not a bill should be put through an expedited process. This type of vote would undoubtedly have different support than a vote for the bill itself, but we are forced to treat them as the same. Additionally, without labels for these votes, we cannot determine how prevalent they are in the data set. This is not ideal, but it is expected that the removal of special votes would increase the homogeneity of each bill's series of votes. If this was in fact the case, our results would be an underestimate of the modeling technique's power.

TABLE III

Top Three Members of the Vote Switching Network Based on an Ensemble of Centrality Measures as Outputted From an ORA Key Entities Report. These Members Seem to be High Ranking but Not Official Party Leaders

Conv.	Name	Party
7	Natalya V. Agafonova	UDAR
7	Brichenko Igor Vitaliyovych	Fatherland
7	Chumak Viktor Vasilyevich	UDAR
8	Illenko Andriy Yuriyovych	No Party
8	Marchenko Alexandr Aleksandrovich	No Party
8	Shurma Igor Mikhailovich	Opposition Bloc

Although the fit model for presidential bills in convocation 7 had real eigenvalues, there were only nine predictable points to fit parameters with. With more bills and more votes, it is possible that the model would take a different form, as it did in convocation 8. Convocation 8 had 96 predictable points from presidential bills and still led to an unstable model, giving more solid evidence that voting behavior is different for presidential bills.

The vote switching network attempts to represent parliamentarians switching their votes due to another parliamentarian's influence in the prior vote. The underlying assumption to this network is that when one MP is strongly influenced by another, they will copy their vote frequently. If a parliamentarian is only convinced to switch their vote for a few important bills, it will be very hard to detect in this network. The procedure also weights votes "against" very heavily, so it is hard to detect any influence spread through absences or votes in favor of bills. In the future work, contextual information about specific votes or parliamentarian relationships may be used to augment information contained in the vote switching network.

VII. Conclusion

We have analyzed a legislative system that differs from those typically studied in the literature, though also appears in Russia, Philippines, and Estonia, to name a few. Under this system, preliminary roll calls are cast on bills. In contrast to prior work focused on individual decision-making, we collapse each vote into a single variable, the percentage of votes in favor of the bill, as it is what determines the bill's success. The sequential revoting on bills leads to a time series of the percentage of votes in favor of each bill, making it a dynamic variable. The traditional tool for modeling dynamic systems is a differential equation. While fields, such as physics, have universal laws to generate differential models, there are no such laws in place for roll call analysis. As such, we predefined our model to be a second-order linear and homogeneous equation and fit its parameters to data using differential evolution.

This model was selected for its simplicity, which has two benefits. First, since legislative dynamics have yet to be studied thoroughly, the simplest possible models should be used in the early stages as to show the area's promise. Second, this specific model guarantees an analytic solution, allowing for an exact fit for the first two votes.

Our modeling process was successfully applied to convocations 7 and 8 of the Rada. Not accounting for sponsoring subject, the models achieved a median error of 7.6% and 6.2% while correctly classifying 77.8% and 85% of votes as passing or failing for convocations 7 and 8, respectively. It was shown that this simple dynamic model is on-par with a neural network approach. Accounting for subject gave modest improvements in performance and highlighted the difference between presidential bills and all others. For both convocations, the only unstable models were those of the presidential bills. This result is in agreement with the expert's suggestions that parliament member's voting behavior changes for presidential bills. Theoretically, the instability in our models suggests that parliamentarians are more influenced by popular opinion on presidential bills. Prior work by Kauder and others suggests that MPs may not be punished for voting against their party but have not looked at whether they could fear repercussions for voting against the president, as we see here. Further investigation into the subject may be appropriate. More practically, this instability also suggests that presidential bills are pushed through the legislature faster, since MPs flip their votes quickly where fewer votes are needed.

The differential model demonstrated the power of the change from the first vote to the second, but lost individual-level information. A vote switching network was created to use this information at the parliamentarian and party level. This network had directed links indicating how often one politician switched their vote to match another's. Analysis of this network found key members of the Rada from UDAR and Fatherland who all became members of the Peter Porchenko Bloc in the following convocation. Potential leaders of Convocation 8 were also found though they were not connected to the presidential party.

We have demonstrated the potential of viewing roll call voting as a dynamic process in two ways, using very simple techniques. We have also demonstrated that these techniques were effective on both convocations 7 and 8 of the Rada, which underwent drastically different political climates. Given the effectiveness of this framing, we believe that this work opens the door to more complex differential models as well as more nuanced vote switching networks. Furthermore, a combined model that accounts for group level and individual level changes simultaneously through vector autoregressive models that may be studied in the future work. Beyond this, future work must include domain knowledge to get at the elusive quarter of votes that seem to be unpredictable through a strictly quantitative model.

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